NMR implementation of a quantum scheduling algorithm

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We have generalized the improved quantum scheduling algorithm proposed by Grover and realized our scheme on a nuclear magnetic resonance (NMR) quantum computer. Experimental results show a good agreement between theory and experiment.

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1 Introduction

The scheduling algorithm solves the intersection problem, i.e., the problem is to find common elements in two sets. Let Alice and Bob be two distant parties who wish to collaborate on a common task. They each have a schedule listing N slots of time. Their schedules can be represented as two stings if Alice and Bob each have N qubits. For each person, the state $|1>$ denotes that a slot is available, and the state $|0>$ denotes that a slot is not available. The intersection problem becomes to find the common 1s in the two strings. Alice and Bob need to exchange information to find the common 1s. The problem is how to reduce the exchanging information [1, 2, 3].

If Alice and Bob exchange classical bits, they will need to exchange $O(N)$ bits. H. Buhrman et al found that Alice and Bob could find the common 1s by exchanging $O(\sqrt{N}\log_2 N)$ qubits, if they used quantum search algorithm [4, 5]. Grover proposed an improved algorithm in theory in the special case when the string of either Alice and Bob has few 1s [6]. Grover's theoretical scheme can be described as follows. Assume that there is a $\log_2 N$ qubit register, by setting $N = 2^n$, where n is an integer. The register can be transmitted between Alice and Bob. Assume that Alice has ϵN 1s in her N bit string, where $\epsilon N \ll N$. For convenience, we assume that Bob also has ϵN 1s in his N bit string, and there is a single common 1 in the two strings. Alice encodes the N slots in the register, by applying the Walsh-Hadamard transform to $|\overline{0}>$, where $|\overline{0}>$ denotes that all qubits in the register lie in state $|0>$. Then, Alice repeats $WI_{\overline{0}}WI_A$ m times, where I_A denotes the suitable inversion for the basis states corresponding to Alice's available slots, $I_{\overline{0}}$ denotes the phase inversion for $|\overline{0}>$, and $m = \pi\sqrt{1/\epsilon}/4$. The register lies in the superposition corresponding to Alice's available slots. The composite transformation used by Alice is denoted by G, represented by $G \equiv (WI_{\overline{0}}WI_A)^m W$. $(GI_{\overline{0}}G^{'}I_B)^m G$ transforms $|\overline{0}>$ to the state corresponding to the available slot of the both of Alice and Bob.

Alice can carry out W, I_A, and $I_{\overline{0}}$. When she needs I_B, she must send the register to Bob. One should notes that the overall state of the register is unaltered during the course of sending [3] . After Bob finishes I_B, he returns the register to Alice. Obviously, the number of times the register needs to be sent to Bob is equal to the number of I_B operations.

In this paper, we generalize Grover's improved algorithm by replacing W by a unitary operator U, and replacing the phase inversions by $\pi/2$ phase rotations, in order to make the algorithm suitable for more cases. Our scheme is implemented using nuclear magnetic resonance (NMR).

2 The generalized Grover's scheme

Our experiments use a sample of carbon-13 labelled chloroform dissolved in d6-acetone. Data are taken at room temperature with a Bruker DRX 500 MHz spectrometer. The resonance frequencies $\nu_1 = 125.76$ MHz for ^{13}C, and

$\nu_2 = 500.13$ MHz for ^1H. The coupling constant J is measured to be 215 Hz. The Hamiltonian of this system is represented as [7]

$$H = -2\pi\nu_1 I_z^1 - 2\pi\nu_2 I_z^2 + 2\pi J I_z^1 I_z^2, \tag{1}$$

by setting $\hbar = 1$, where $I_z^k (k = 1, 2)$ are the matrices for $\hat{z}$-component of the angular momentum of the spins. The evolution caused by a radio-frequency (rf) pulse on resonance along x or $-y$ axis is represented as $R_x^k(\varphi)$ or $R_y^k(-\varphi)$, with k specifying the affected spin. The pulse used above is denoted by $[\varphi]_x^k$ or $[-\varphi]_y^k$. The coupled-spin evolution is denoted as

$$[\tau] = e^{-i2\pi J\tau I_z^1 I_z^2}. \tag{2}$$

The pseudo-pure state

$$\rho_0 = I_z^1/2 + I_z^2/2 + I_z^1 I_z^2. \tag{3}$$

is prepared by using spatial averaging [10, 11]. ρ_0 is equivalent to $|00>$ in our experiments, where $|0>$ denotes up spin state.

The Walsh-Hadamard transform is replaced by $U = R_y^1(\pi/2)R_y^2(-\pi/2)$, which is represented as

$$U = \frac{1}{2}\begin{pmatrix} 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & -1 \\ -1 & -1 & 1 & 1 \end{pmatrix}. \tag{4}$$

Alice and Bob each have a four bit string, in which the positions of slots are encoded by a two qubit register. The basis states $|00>$, $|01>$, $|10>$, $|11>$ correspond to the first, second, third, and fourth slots, respectively. Assume that Alice's available slots are the first and the second ones, which correspond to $|00>$, and $|01>$, respectively. I_A is replaced by a $\pi/2$ phase rotation [12], represented as

$$I_{A12}^{\frac{\pi}{2}} = \begin{pmatrix} i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{5}$$

Considering the phase matching condition [13], $I_{\overline{0}}$ is replaced by

$$I_{0}^{\frac{\pi}{2}} = \begin{pmatrix} i & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{6}$$

We assume that there are three cases for Bob's available slots: 1) The first and the fourth slots are available; 2) The second and the third slots are available; and 3) The first and the second slots are available. For the three cases, I_B is replaced by

$$I_{B14}^{\frac{\pi}{2}} = \begin{pmatrix} i & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & i \end{pmatrix}, \tag{7}$$

$$I_{B23}^{\frac{\pi}{2}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{8}$$

$$I_{B12}^{\frac{\pi}{2}} = \begin{pmatrix} i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{9}$$

respectively. By defining the composite operators $G_{12} \equiv -UI_{0}^{\frac{\pi}{2}}U^{-1}I_{A12}^{\frac{\pi}{2}}U$, $Q_1 \equiv -G_{12}I_{0}^{\frac{\pi}{2}}G_{12}^{-1}I_{B14}^{\frac{\pi}{2}}G_{12}$, $Q_2 \equiv -G_{12}I_{0}^{\frac{\pi}{2}}G_{12}^{-1}I_{B23}^{\frac{\pi}{2}}G_{12}$, and $Q_{12} \equiv -G_{12}I_{0}^{\frac{\pi}{2}}G_{12}^{-1}I_{B12}^{\frac{\pi}{2}}G_{12}$, we obtain

$$G_{12}|00>= e^{-i\pi/4}(|00> +|01>)/\sqrt{2}, \tag{10}$$

$$Q_1|00>= -i|00>, \tag{11}$$

$$Q_2|00>= -i|01>, \tag{12}$$

$$Q_{12}|00>= e^{-i\pi/4}(|00> +|01>)/\sqrt{2}. \tag{13}$$

Similarly, if Alice's available slots are the third and fourth ones, we replace $I^{\frac{\pi}{2}}_{A12}$ by

$$I^{\frac{\pi}{2}}_{A34} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \end{pmatrix} \tag{14}$$

and obtain

$$G_{34}|00>= -e^{-i\pi/4}(|10> +|11>)/\sqrt{2}, \tag{15}$$

where $G_{34} \equiv -UI^{\frac{\pi}{2}}_{0}U^{-1}I^{\frac{\pi}{2}}_{A34}U$. We also assume that there are three cases for Bob: 1) The first and the fourth slots are available; 2) The second and the third slots are available; and 3) The third and the fourth slots are available. By defining $Q_4 \equiv -G_{34}I^{\frac{\pi}{2}}_{0}G^{-1}_{34}I^{\frac{\pi}{2}}_{B14}G_{34}$, $Q_3 \equiv -G_{34}I^{\frac{\pi}{2}}_{0}G^{-1}_{34}I^{\frac{\pi}{2}}_{B23}G_{34}$, and $Q_{34} \equiv -G_{34}I^{\frac{\pi}{2}}_{0}G^{-1}_{34}I^{\frac{\pi}{2}}_{B34}G_{34}$, we obtain

$$Q_4|00>= i|11>, \tag{16}$$

$$Q_3|00>= i|10>, \tag{17}$$

$$Q_{34}|00>= -e^{-i\pi/4}(|10> +|11>)/\sqrt{2}, \tag{18}$$

where

$$I^{\frac{\pi}{2}}_{B34} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \end{pmatrix}. \tag{19}$$

The overall phases before wave functions can be ignored.

3 Experimental procedure and results

The coupled-spin evolution described as Eq.(2) is realized by pulse sequence $\tau/2 - [\pi]_x^{1,2} - \tau/2 - [-\pi]_x^{1,2}$ [16], where $[\pi]_x^{1,2}$ denotes a nonselective pulse (hard pulse). $[\pi]_x^{1,2}$ pulses are applied in pairs each of which take opposite phases in order to reduce the error accumulation causes by imperfect calibration of π-pulses [15]. U is realized by $[\pi/2]_y^1 - [-\pi/2]_y^2$. $I_{A12}^{\frac{\pi}{2}} = R_y^1(\pi/2)R_x^1(\pi/2)R_y^1(-\pi/2)$ (up to an irrelevant overall phase factor), and is realized by $[-\pi/2]_y^1 - [\pi/2]_x^1 - [\pi/2]_y^1$, noting that the pulses are applied from left to right. $I_{A34}^{\frac{\pi}{2}} = R_y^1(\pi/2)R_x^1(-\pi/2)R_y^1(-\pi/2)$. By modifying the pulse sequences in Ref. [17, 13], we find $I_0^{\frac{\pi}{2}} = R_y^{1,2}(\pi/2)R_x^{1,2}(\pi/4)R_y^{1,2}(-\pi/2)[15/4J]$. By optimizing the pulse sequences, we obtain

$G_{A12} = -R_y^1(\pi/2)R_y^1(\pi/2)R_x^{1,2}(\pi/4)R_y^{1,2}(-\pi/2)[15/4J]R_x^1(\pi/2)$,
$G_{A34} = -R_y^1(\pi/2)R_y^1(\pi/2)R_x^{1,2}(\pi/4)R_y^{1,2}(-\pi/2)[15/4J]R_x^1(-\pi/2)$.
$G_{A12}^{-1} = -R_x^1(-\pi/2)[1/4J]R_y^{1,2}(\pi/2)R_x^{1,2}(-\pi/4)R_y^1(-\pi/2)R_y^1(-\pi/2)$;
$G_{A34}^{-1} = -R_x^1(\pi/2)[1/4J]R_y^{1,2}(\pi/2)R_x^{1,2}(-\pi/4)R_y^1(-\pi/2)R_y^1(-\pi/2)$. Moreover, $I_{B14}^{\frac{\pi}{2}} = [7/2J]$, $I_{B23}^{\frac{\pi}{2}} = [1/2J]$, $I_{B12}^{\frac{\pi}{2}} = I_{A12}^{\frac{\pi}{2}}$, and $I_{B34}^{\frac{\pi}{2}} = I_{A34}^{\frac{\pi}{2}}$.

All experiments are acquired in an identical fashion in order that the relative phases of signals in the spectra are meaningful[18]. We first prepare pseudo- pure state $|00>$. By applying $[\pi]_x^2$, $[\pi]_x^1$, and $[\pi]_x^{1,2}$ to $|00>$, the other pseudo- pure states $|01>$, $|10>$, and $|11>$ are obtained, respectively. Figs.1(a-d) are corresponding to the four pseudo- pure states $|00>$, $|01>$, $|10>$, and $|11>$. In each figure, the main spectrum represents the photon spectrum obtained through a readout pulse $[\pi/2]_y^2$ and the inset represents the carbon spectrum obtained through $[\pi/2]_y^1$.

The scheduling algorithm starts with pseudo-pure state $|00>$. Q_1, Q_2, Q_3, and Q_4 transform $|00>$ into $|00>$, $|01>$, $|10>$, and $|11>$, respectively. The photon spectra are shown as the main figures in Figs.2(a-d) through readout pulse $[\pi/2]_y^2$, and the carbon spectra are shown as the insets in Figs.2(a-d) through $[\pi/2]_y^1$, corresponding to Q_1, Q_2, Q_3, and Q_4, respectively. By comparing Figs.2 (a-d) with Figs.1 (a-d), respectively, we corroborate that the system lies in $|00>$, $|01>$, $|10>$, and $|11>$, respectively, after the completion of the algorithm.

4 Conclusion

Alice and Bob each have two available slots in their schedules either of which consists of four slots. They can find the common slots by exchanging the register for only one time. By generalizing Grover's original theoretical scheme for scheduling algorithm, we have realized the algorithm on a two qubit NMR quantum computer, and nontrivial results are demonstrated.

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Figure Captions

1. The photon spectra shown as main figures and the carbon spectra shown as smaller insets, when the two-spin system lies in various pseudo-pure states via readout pulses selective for ^1H denoted by $[\pi/2]_y^2$ (main figures) and for ^{13}C denoted by $[\pi/2]_y^1$ (smaller insets). (a-d) are spectra corresponding to pseudo- pure states $|00>$, $|01>$, $|10>$ and $|11>$, respectively.

2. The photon spectra shown as main figures and the carbon spectra shown as smaller insets via $[\pi/2]_y^2$ (main figures) and $[\pi/2]_y^1$ (smaller insets) after the completion of the scheduling algorithm. Q_1, Q_2, Q_3, and Q_4 transform $|00>$ into $|00>$, $|01>$, $|10>$, and $|11>$, respectively, corresponding to (a-d). By comparing Figs.2 (a-d) with Figs.1 (a-d), respectively, we corroborate the algorithm has been implemented.

[Figure 1 about here.]

[Figure 2 about here.]

List of Figures

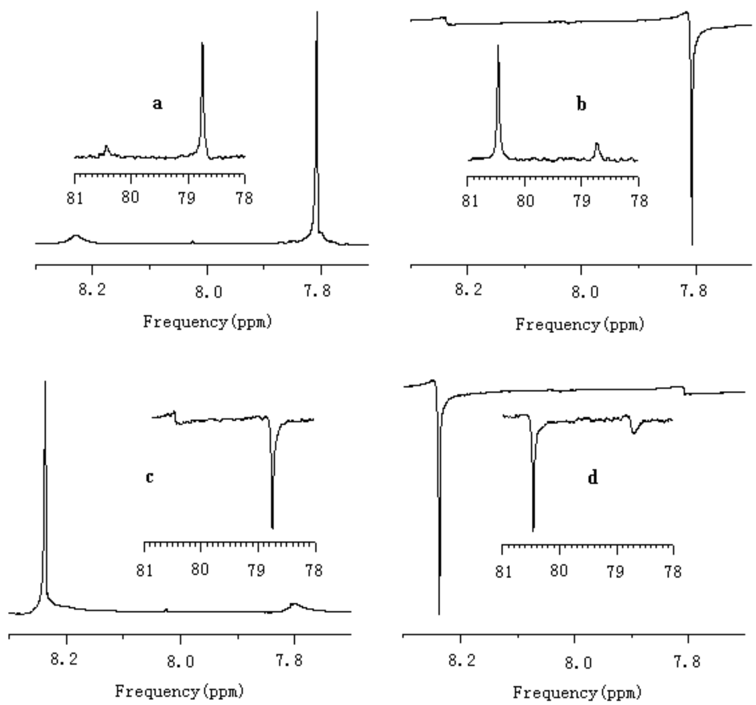

Fig.1

1

Figure 1:

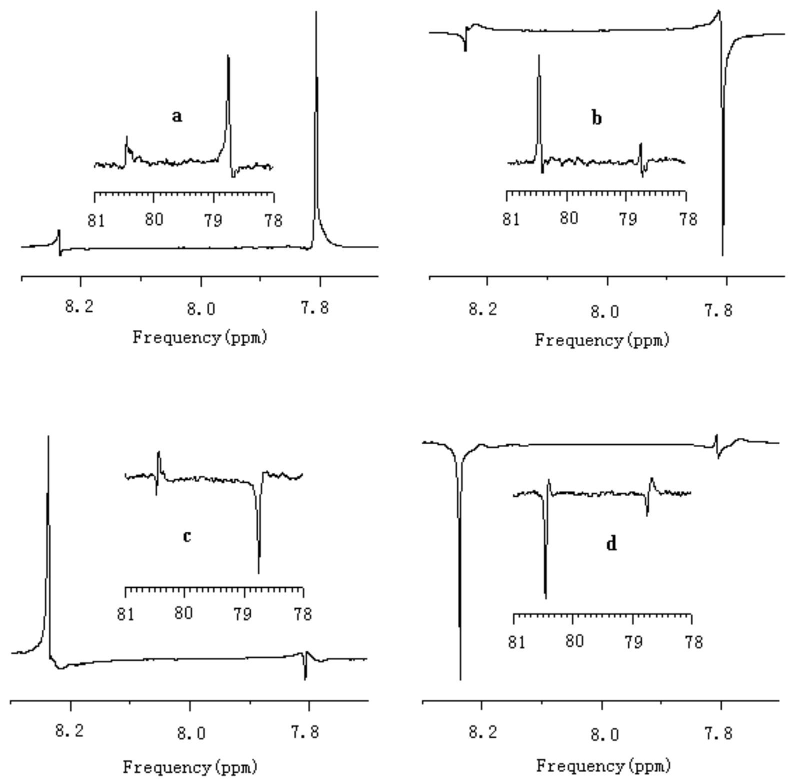

Fig2

2

Figure 2: